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LETTER TO THE EDITOR

Stability of critical behaviour of weakly disordered systems to introduction of potentials with replica symmetry breaking

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Abstract

A field-theoretic description of the critical behaviour of the weakly disordered systems with p-component order parameter is given. Directly, for three-dimensional systems a renormalization analysis of the effective replicated Hamiltonian of a model with replica symmetry breaking (RSB) potentials is carried out in the two-loop approximation. For the case with first-step RSB the fixed points (FPs) corresponding to stability of the various types of critical behaviour are identified with the use of the Padé–Borel summation technique. Analysis of FPs has shown a stability of the critical behaviour of the weakly disordered systems with respect to RSB effects and realization of the former scenario of disorder influence on critical behaviour.

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The effects produced by weak quenched disorder on critical phenomena have been studied for many years [1–5]. According to the Harris criterion [1], the disorder affects the critical behaviour only if α, the specific heat exponent of the pure system, is positive. In this case a new universal critical behaviour, with new critical exponents, is established. In contrast, when $\alpha < 0$, the disorder appears to be irrelevant for the critical behaviour.

In dealing with the weak quenched disorder the traditional approach is the replica method [4, 5], and in terms of replicas all the results obtained for the disorder systems correspond to the so-called replica-symmetric (RS) solutions. Physically this means that only a unique ground state is assumed to be relevant for the observable thermodynamics. However, in a number of papers [6–8] ideas about replica symmetry breaking (RSB) in the systems with quenched disorder were presented. For the first time in [6] physical arguments showing that in the presence of the quenched disorder there exist numerous local minimal-energy configurations separated by finite barriers and a demonstration of how the summation over these local minimum configurations can provide additional RSB interaction potentials for fluctuating fields were offered. The renormalization group (RG) description of the classical ϕ^4 model with

RSB potentials was presented in the one-loop approximation using ε-expansion [6–8]. It was shown that the RSB degrees of freedom produce a dramatic effect on the asymptotic behaviour of the RG flows, such that for a general type of RSB there exist no stable fixed points (FPs), and RG equations arrive in the strong-coupling regime.

However, our numerous investigations of pure and disordered systems performed in the two-loop and higher orders of the approximation for the three-dimensional system directly together with methods of series summation show that the predictions made in the lowest order of the approximation, especially on the basis of the ε-expansion, can differ strongly from the real critical behaviour [9]. Therefore, the results of RSB effect investigation in [6–8] must be reconsidered with the use of a more accurate field-theoretic approach in the higher orders of the approximation.

In this Letter, we realize the field-theoretical RG description in the two-loop approximation of the three-dimensional model of the weakly disordered systems with RSB interaction potentials of forth order on fluctuating fields. We carry out the solution of the RG equations with the use of the series summation method and the analysis of stability of various types of critical behaviour with respect to RSB effects.

We consider an O(p)-symmetric Ginzburg–Landau–Wilson model of a spin system with weak quenched disorder near the critical point given by the Hamiltonian

$$H = \int \mathrm{d}^d x \left\{ \frac{1}{2} \sum_{i=1}^{p} [\nabla \phi_i(x)]^2 + \frac{1}{2}[\tau - \delta\tau(x)] \sum_{i=1}^{p} \phi_i^2(x) + \frac{1}{4} g \sum_{i,j=1}^{p} \phi_i^2(x)\phi_j^2(x) \right\} \tag{1}$$

where $\phi_i(x)$ is the p-component order parameter and $\delta\tau(x)$ is the Gaussian-distributed random transition temperature with the second moment of distribution $\langle\langle(\delta\tau(x))^2\rangle\rangle \sim u$ defined by the positive constant u, which is proportional to the concentration of defects. The use of the standard replica trick gives us the possibility to easily average over the disorder and reduce the task of statistical description of the weakly disordered system with the Hamiltonian (1) to the homogeneous system with the effective Hamiltonian

$$H_n = \int \mathrm{d}^d x \left\{ \frac{1}{2} \sum_{i=1}^{p} \sum_{a=1}^{n} [[\nabla \phi_i^a(x)]^2 + \tau[\phi_i^a(x)]^2] + \frac{1}{4} \sum_{i,j=1}^{p} \sum_{a,b=1}^{n} g_{ab}[\phi_i^a(x)]^2[\phi_j^b(x)]^2 \right\} \tag{2}$$

which is a functional of n replications of the original order parameter with an additional vertex u in the RS matrix $g_{ab} = g\delta_{ab} - u$. The properties of the original disordered system are obtained in the replica number limit $n \to 0$. The following standard RG procedure based on the statistical calculation of the contribution to the partition function of long-wavelength order parameter fluctuations around the global minimum state with $\phi(x) = 0$ gives us the possibility to find the various types of critical behaviour and conditions of their stability and carry out the calculation of critical exponents.

However, as shown in [6–8], the fluctuations of random transition temperature $\delta\tau(x)$ for $[\tau - \delta\tau(x)] < 0$ can lead to realization in a system of numerous regions with $\phi(x) \neq 0$ displayed through the numerous local minimal-energy configurations separated from the ground state by finite barriers. In this case the direct application of the traditional RS RG scheme may be questioned. For statistical description of such systems near the phase transition point the Parisi RSB scheme was used in [6–8] by analogy with spin glasses [10]. It was argued that spontaneous RSB can occur due to the interaction of the fluctuating fields with the local non-perturbative degrees of freedom from the multiple-local-minimum solutions of the mean-field equations. It was shown that the summation over these solutions in the replica partition function can provide the additional non-trivial RSB potential $\sum_{a,b} g_{ab}\phi_a^2\phi_b^2$ in which the matrix g_{ab} has the Parisi RSB structure [10]. According to the technique of the Parisi RSB algebra, in the limit $n \to 0$ the matrix g_{ab} is parametrized in terms of its diagonal elements $\tilde{g}$ and the off-diagonal function $g(x)$ defined in the interval $0 < x < 1$: $g_{ab} \to (\tilde{g}, g(x))$. The operations with the matrices g_{ab} are given by the following rules:

$$g_{ab}^k \to (\tilde{g}^k; g^k(x)), (\hat{g}^2)_{ab} = \sum_{c=1}^n g_{ac}g_{cb} \to (\tilde{c}; c(x)), (\hat{g}^3)_{ab} = \sum_{c,d=1}^n g_{ac}g_{cd}g_{db} \to (\tilde{d}; d(x)) \tag{3}$$

where

$$\begin{aligned}\tilde{c} &= \tilde{g}^2 - \int_0^1 \mathrm{d}x\, g^2(x)\\ c(x) &= 2\left[\tilde{g} - \int_0^1 \mathrm{d}y\, g(y)\right]g(x) - \int_0^x \mathrm{d}y\,[g(x) - g(y)]^2\end{aligned} \tag{4}$$

$$\begin{aligned}\tilde{d} &= \tilde{c}\tilde{g} - \int_0^1 \mathrm{d}x\, c(x)g(x)\\ d(x) &= \left[\tilde{g} - \int_0^1 \mathrm{d}y\, g(y)\right]c(x) + \left[\tilde{c} - \int_0^1 \mathrm{d}y\, c(y)\right]g(x)\\ &\quad - \int_0^x \mathrm{d}y\,[g(x) - g(y)][c(x) - c(y)].\end{aligned} \tag{5}$$

The RS situation corresponds to the case $g(x) = \text{const}$ independent of x.

We carried out the field-theoretical RG description of the three-dimensional model with the effective replicated Hamiltonian (2) in which the matrix g_{ab} has the RSB structure in the two-loop approximation. In the field-theoretic approach the asymptotic critical behaviour of systems in the fluctuation region is determined by the Callan–Symanzik RG equation for the vertex parts of the irreducible Green functions. To calculate the β functions as functions of the renormalized elements of the matrix g_{ab} appearing in the RG equation, we used the method based on the Feynman diagram technique and the renormalization procedure [11]. We obtained the following expressions for the two-point vertex function $\Gamma^{(2)}$ and four-point vertex functions $\Gamma^{(4)}_{ab}$:

$$\left.\frac{\partial \Gamma^{(2)}}{\partial k^2}\right|_{k^2=0} = 1 + 4fg_{aa}^2 + 2pf\sum_{c=1}^n g_{ac}g_{ca} \tag{6}$$

$$\begin{aligned}\Gamma^{(4)}_{ab}|_{k_i=0} = g_{ab} &- p\sum_{c=1}^n g_{ac}g_{cb} - 4g_{aa}g_{ab} - 4g_{ab}^2 + (8+16h)g_{ab}^3 + (24+8h)g_{aa}^2 g_{ab}\\ &+48hg_{aa}g_{ab}^2 + 4g_{aa}g_{bb}g_{ab} + 8ph\sum_{c=1}^n g_{ac}g_{cb}^2 + 8phg_{ab}\sum_{c=1}^n g_{ac}g_{cb}\\ &+4phg_{ab}\sum_{c=1}^n g_{ac}^2 + 2p\sum_{c=1}^n g_{ac}g_{cc}g_{cb}\\ &+4pg_{aa}\sum_{c=1}^n g_{ac}g_{cb} + p^2\sum_{c,d=1}^n g_{ac}g_{cd}g_{db}\end{aligned} \tag{7}$$

where

$$f(d) = -\frac{1}{J^2}\frac{\partial}{\partial k^2}\int \left.\frac{\mathrm{d}^d k_1\,\mathrm{d}^d k_2}{(k_1^2+1)(k_2^2+1)((k_1+k_2+k)^2+1)}\right|_{k^2=0} \tag{8}$$

$$h(d) = \frac{1}{J^2}\int \frac{\mathrm{d}^d k_1\,\mathrm{d}^d k_2}{(k_1^2+1)^2(k_2^2+1)((k_1+k_2)^2+1)} \tag{9}$$

$$J = \int \mathrm{d}^d k/(k^2+1)^2 \qquad f(d=3) = \tfrac{2}{27} \qquad h(d=3) = \tfrac{2}{3} \tag{10}$$

and made the redefinition $g_{ab} \to g_{ab}/J$. However, the following renormalization procedure for vertex functions is made difficult because of complicated expressions (3)–(4) for the operations with the matrices g_{ab}. The steplike structure of the function $g(x)$ revealed in [6–8] gives us the possibility to realize the renormalization procedure. In this Letter we considered only the matrices g_{ab} which have the structure known as the one-step RSB with function $g(x)$ of the next view:

$$g(x) = \begin{cases} g_0 & \text{for} \quad 0 \leqslant x < x_0 \\ g_1 & \text{for} \quad x_0 < x \leqslant 1 \end{cases} \tag{11}$$

where $0 \leqslant x_0 \leqslant 1$ is the coordinate of the step and it remains an arbitrary parameter. The value of x_0 is not changed during the renormalization procedure and remains the same as in the starting function $g_0(x)$. In consequence the RG transformations of the effective replicated Hamiltonian with RSB potentials are determined by the three parameters $\tilde{g}$, g_0 and g_1. We obtained the β functions in the two-loop approximation in the form of the expansion series in renormalized parameters $\tilde{g}$, g_0 and g_1:

$$\begin{aligned}
\beta_1 &= -\tilde{g} + (p+8)\tilde{g}^2 - px_0{g_0}^2 - p(1-x_0){g_1}^2 - \tfrac{4}{27}(41p+190)\tilde{g}^3 \\
&\quad + \tfrac{92}{27}\, px_0\tilde{g}{g_0}^2 + \tfrac{92}{27}\, p(1-x_0)\tilde{g}{g_1}^2 - \tfrac{8}{3}\, px_0{g_0}^3 - \tfrac{8}{3}\, p(1-x_0){g_1}^3 \\
\beta_2 &= -g_0 - (4-2px_0){g_0}^2 + (4+2p)\tilde{g}g_0 + 2p(1-x_0)g_0g_1 \\
&\quad + \tfrac{16}{3}\,(\tfrac{77}{36}\, px_0 - 1){g_0}^3 - \tfrac{92}{27}\,(p+2)\tilde{g}^2 g_0 - \tfrac{8}{3}\,(2px_0 - 5p - 6)\tilde{g}{g_0}^2 \\
&\quad + \tfrac{40}{3}\, p(1-x_0){g_0}^2 g_1 - \tfrac{52}{27}\, p(1-x_0)g_0{g_1}^2 - \tfrac{16}{3}\, p(1-x_0)\tilde{g}g_0g_1 \\
\beta_3 &= -g_1 - (px_0 - 2p + 4){g_1}^2 + px_0{g_0}^2 + (4+2p)\tilde{g}g_1 \\
&\quad - (\tfrac{92}{27}\, px_0 - \tfrac{308}{27}\, p + \tfrac{16}{3}){g_1}^3 + \tfrac{8}{3}\, px_0{g_0}^3 - \tfrac{92}{27}\,(p+2)\tilde{g}^2 g_1 \\
&\quad - \tfrac{8}{3}\, px_0\tilde{g}{g_0}^2 + (\tfrac{8}{3}\, px_0 + 8p + 16)\tilde{g}{g_1}^2 + \tfrac{20}{27}\, px_0{g_0}^2 g_1 .
\end{aligned} \tag{12}$$

By analogy with papers [6–8] we changed $g_{a\neq b} \to -g_{a\neq b}$ in the expressions (12) for the β functions, so that the off-diagonal elements $g_{a\neq b}$ would be positively defined.

It is well known that perturbation series are asymptotic series, and that the vertices describing the interaction of the order parameter fluctuations in the fluctuating region $\tau \to 0$ are large enough that expressions (12) cannot be used directly. For this reason, to extract the required physical information from the obtained expressions, we employed the Padé–Borel approximation of the summation of asymptotic series extended to the multiparameter case. The direct and inverse Borel transformations for the multiparameter case have the form

$$\begin{aligned}
f(\tilde{g}, g_0, g_1) &= \sum_{i,j,k} c_{ijk}\tilde{g}^i g_0^j g_1^k = \int_0^\infty \mathrm{e}^{-t} F(\tilde{g}t, g_0t, g_1t)\,\mathrm{d}t \\
F(\tilde{g}, g_0, g_1) &= \sum_{i,j,k} \frac{c_{ijk}}{(i+j+k)!}\tilde{g}^i g_0^j g_1^k .
\end{aligned} \tag{13}$$

A series in the auxiliary variable θ is introduced for analytical continuation of the Borel transform of the function

$$\tilde{F}(\tilde{g}, g_0, g_1, \theta) = \sum_{k=0}^{\infty} \theta^k \sum_{i=0}^{k} \sum_{j=0}^{k-i} \frac{c_{i,j,k-i-j}}{k!}\tilde{g}^i g_0^j g_1^{k-i-j} \tag{14}$$

to which the [L/M] Padé approximation is applied at the point $\theta = 1$. To perform the analytical continuation, the Padé approximant of [L/1] type may be used, which is known to provide rather good results for various Landau–Wilson models (see, e.g. [12,13]). The property of preserving

Table 1. Coordinates of the FPs and eigenvalues of the stability matrix for $p = 1$.

Type	x_0	$\tilde{g}^*$	g_0^*	g_1^*	λ_1	λ_2	λ_3
1		0.177 4103	0	0	0.653 55	−0.169 24	−0.169 24
2		0.184 3726	0.081 2240	0.081 2240	0.525 ± 0.089i		0.211
3	0.0	0.184 3726	0	0.081 2240	0.525 319 ± 0.089 273i		−0.039 167
	0.1	0.183 9722	0	0.082 9404	0.535 185 ± 0.098 291i		−0.049 185
	0.2	0.183 5134	0	0.084 6432	0.547 065 ± 0.106 665i		−0.059 851
	0.3	0.182 9917	0	0.086 3186	0.560 666 ± 0.113 305i		−0.071 187
	0.4	0.182 4035	0	0.087 9503	0.576 473 ± 0.118 038i		−0.083 210
	0.5	0.181 7458	0	0.089 5200	0.595 060 ± 0.120 271i		−0.095 927
	0.6	0.181 0165	0	0.091 0067	0.617 241 ± 0.118 872i		−0.109 334
	0.7	0.180 2154	0	0.092 3872	0.643 936 ± 0.111 389i		−0.123 415
	0.8	0.179 3442	0	0.093 6384	0.675 972 ± 0.092 079i		−0.138 133
	0.9	0.178 4070	0	0.094 7426	0.713 456 ± 0.035 266i		−0.153 431
	1.0	0.177 4103	0	0.095 6920	0.857 325	0.653 55	−0.169 237

Table 2. Coordinates of the FPs and eigenvalues of the stability matrix for $p = 2$.

Type	x_0	$\tilde{g}^*$	g_0^*	g_1^*	λ_1	λ_2	λ_3
1		0.155 8303	0	0	0.667 315	−0.001 672	−0.001 672
2		0.155 8310	0.000 5837	0.000 5837	0.667 312	0.001 682	0.000 004
3	0.0	0.155 8310	0	0.000 5837	0.667 313	0.001 683	−0.000 001
	0.1	0.155 8310	0	0.000 6143	0.667 313	0.001 684	−0.000 088
	0.2	0.155 8310	0	0.000 6483	0.667 313	0.001 685	−0.000 186
	0.3	0.155 8310	0	0.000 6863	0.667 313	0.001 686	−0.000 296
	0.4	0.155 8310	0	0.000 7291	0.667 313	0.001 687	−0.000 419
	0.5	0.155 8310	0	0.000 7775	0.667 313	0.001 687	−0.000 559
	0.6	0.155 8309	0	0.000 8327	0.667 313	0.001 688	−0.000 717
	0.7	0.155 8308	0	0.000 8964	0.667 314	0.001 690	−0.000 901
	0.8	0.155 8307	0	0.000 9707	0.667 314	0.001 692	−0.001 116
	0.9	0.155 8306	0	0.001 0583	0.667 315	0.001 694	−0.001 369
	1.0	0.155 8303	0	0.001 1633	0.667 316	0.001 696	−0.001 672

the symmetry of a system during application of the Padé approximation by the θ method, as in [12], has become important for multivertex models. We used the [2/1] approximant to calculate the β functions in the two-loop approximation.

The nature of the critical behaviour is determined by the existence of a stable FP satisfying the system of equations

$$\beta_k(\tilde{g}^*, g_0^*, g_1^*) = 0 \qquad (k = 1, 2, 3). \tag{15}$$

We have found three types of non-trivial FP in the physical region of parameter space $\tilde{g}^*, g_0^*, g_1^* \geqslant 0$ for different values of $p = 1, 2, 3$, which are presented in tables 1–3 (the exception was made in the case with $p = 3$: when presented table 3 the coordinates of type II and III FPs are characterized by unphysical negative values of g_0^* and g_1^*). Type I with $\tilde{g}^* \neq 0$, $g_0^* = g_1^* = 0$ corresponds to the RS FP of a pure system, type II with $\tilde{g}^* \neq 0$, $g_0^* = g_1^* \neq 0$ is a disorder-induced RS FP and type III with $\tilde{g}^* \neq 0$, $g_0^* = 0$, $g_1^* \neq 0$ corresponds to the one-step RSB FP. The values of parameters $\tilde{g}^*$, g_1^* for the one-step RSB FP depend on the coordinate of the step x_0, and we present in tables 1–3 the received values of these parameters in the range $0 \leqslant x_0 \leqslant 1$ with changes through the step $\Delta x_0 = 0.1$.

Table 3. Coordinates of the FPs and eigenvalues of the stability matrix for $p = 3$.

Type	x_0	$\tilde{g}^*$	g_0^*	g_1^*	λ_1	λ_2	λ_3
1		0.138 2700	0	0	0.681 378	0.131 537	0.131 537
2		0.141 9323	−0.035 8629	−0.035 8629	0.672 676	−0.089 135	−0.005 783
3	0.0	0.141 9323	0	−0.035 8629	0.672 676	−0.089 135	−0.005 783
	0.1	0.141 9931	0	−0.038 1865	0.672 729	−0.086 515	0.001 104
	0.2	0.142 0386	0	−0.040 8334	0.672 845	−0.083 560	0.008 802
	0.3	0.142 0600	0	−0.043 8761	0.673 046	−0.080 206	0.017 469
	0.4	0.142 0441	0	−0.047 4104	0.673 361	−0.076 366	0.027 299
	0.5	0.141 9699	0	−0.051 5650	0.673 831	−0.071 931	0.038 540
	0.6	0.141 8040	0	−0.056 5177	0.674 509	−0.066 754	0.051 518
	0.7	0.141 4913	0	−0.062 5193	0.675 476	−0.060 636	0.066 656
	0.8	0.140 9374	0	−0.069 9349	0.676 837	−0.053 299	0.084 517
	0.9	0.139 9720	0	−0.079 3131	0.678 742	−0.044 336	0.105 835
	1.0	0.138 2700	0	−0.091 5089	0.681 378	−0.033 124	0.131 537

The type of critical behaviour of this disordered system for each value of p is determined by the stability of the corresponding FP. The requirement that the FP be stable reduces to the condition that the eigenvalues λ_i of the matrix

$$B_{i,j} = \frac{\partial \beta_i(\tilde{g}^*, g_0^*, g_1^*)}{\partial g_j} \tag{16}$$

lie in the right-hand side complex half-plane.

Analysis of the values λ_i for FPs presented in tables 1–3 shows that for the Ising model ($p = 1$) and the XY model ($p = 2$) the disorder-induced RS FPs are stable. However, we believe that in the higher field-theoretic orders of approximation the RS FP of a pure system will be stable for the XY model. Two facts indicate this: the weak stability of the disorder-induced RS FP and that in the two-loop approximation the marginal value of $p_c = 2.0114$ for the borderline between regions of stability for the disorder-induced RS FP and the RS FP of a pure system. In the higher orders of approximation the marginal value of $p_c < 2$, such as the specific heat exponent $\alpha > 0$ for the pure XY model. For the Heisenberg model ($p = 3$) the RS FP of a pure system is stable and both other types of FP are characterized by unphysical values of coordinates g_0^* and g_1^*.

The obtained RS FP values for vertices $\tilde{g}$, g_0 and the eigenvalues λ_1 and λ_2 of the stability matrix correspond to results of paper [14], in which a field-theoretic treatment of disordered three-dimensional spin systems was presented in the two-loop approximation. The vertices v_1 and v_2 introduced in [14] are connected with the vertices $\tilde{g}$ and g_0 by the relations $v_1 = (p + 8)(\tilde{g} + g_0) + 9g_0$ and $v_2 = 8g_0$. We have calculated the static critical exponents from the γ functions in the corresponding stable RS FPs resummed by the generalized Padé–Borel method (table 4). For comparison we also present in table 4 values of the critical exponents from [15, 16] received for pure and disordered three-dimensional systems without RSB in the six-loop approximation. Comparison of the exponent values shows that their differences are not more than 0.01. This gives us the possibility to consider our results of the RSB effect investigation as reliable. The model with RSB potentials is another example of the multivertex models [12] for which the predictions made on the basis of the ε-expansion can differ strongly from results of the use of a more accurate field-theoretic approach for the three-dimensional system directly together with methods of series summation. This situation is explained by the competition of numerous types of FP in the parameter space of the multivertex models. Therefore, the spread of results of the ε-expansion from $\varepsilon \ll 1$ to

Table 4. Critical exponents of the three-dimensional models for RS FPs.

Model	FP	η	ν	γ	β	α
Ising	RS1 this work	0.028	0.631	1.244	0.324	0.107
	[15]	0.031(4)	0.630(2)	1.241(2)	0.325(2)	0.110(5)
	RS2 this work	0.028	0.672	1.329	0.345	−0.015
	[16]	0.030(3)	0.678(10)	1.330(17)	0.349(5)	−0.034(30)
XY	RS1 this work	0.029	0.667	1.318	0.34	−0.001
	[15]	0.034(3)	0.669(1)	1.316(1)	0.346(1)	−0.007(6)
Heisenberg	RS1 this work	0.028	0.697	1.379	0.369	−0.092
	[15]	0.034(3)	0.705(1)	1.387(1)	0.364(1)	−0.115(9)

$\varepsilon = 1$ is impossible, as a rule, without intersection of the stability ranges for the various types of FP.

Thus, the RG investigations carried out in the two-loop approximation show the stability of the critical behaviour of weakly disordered three-dimensional systems with respect to the RSB effects. In dilute Ising-like systems the disorder-induced critical behaviour is realized with a RS FP. The weak disorder is irrelevant for the critical behaviour of systems with a multicomponent order parameter although the proof for systems with a two-component order parameter demands calculations in the higher orders of approximation. The possible influence of the RSB degrees of freedom on the critical behaviour of highly disordered systems can be nonperturbatively revealed by the use of the Monte Carlo simulation method [17] for definition of the probability distributions for the order parameter and random transition temperature fluctuations. At the present time our group is carrying out the Monte Carlo simulation of the disordered three-dimensional Ising model to check this possibility.

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